

## Introduction to basic SMath Studio operation

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In this document we introduce the use of SMath Studio for the purpose of calculating mathematical expressions, and solving algebraic equations.

### 1. Where to download SMath Studio?

(These instructions are valid as of August 05, 2011)

Download SMath Studio by following this download page link:

[http://en.smath.info/forum/yaf\\_topics12\\_Download-SMath-Studio.aspx](http://en.smath.info/forum/yaf_topics12_Download-SMath-Studio.aspx)

\* Follow the link below the title "Download SMath Studio" for the most recent version of the software. Then scroll down until you find the download link for your operating system (e.g., Desktop Windows) under the heading "Free Download"

\* For Windows select either:

- "SMathStudioDesktop.0\_89\_8.Setup.msi" to install in your computer, or
- "SMathStudioDesktop.0\_89\_8.Portable.exe" to carry in a portable drive

Try the Tutorials - developed by yours truly:

<http://smath.info/wiki/Tutorials.ashx>

Study these Examples for applications:

<http://smath.info/wiki/Examples.ashx>

Follow postings in the Wiki - information and discussion on the software:

<http://smath.info/wiki/AllPages.aspx>

These instructions are also available at my SMath Studio web site:

<http://www.neng.usu.edu/cee/faculty/gurro/SMathStudio.html>

### 2. Basic operation of the SMath Studio worksheet

\* To turn paper grid on/off: View>Grid

\* To select the number of decimals to display in results: Tools>Options  
Set "Decimal places", for example, to 4

\* To enter text: click at the point where you want to start, and type  
" followed by your text - Example: In the space immediately below,  
click " and then type:

My first attempt at text in SMath Studio.

Enter your  
solution here ->

- \* To enter a numerical calculation: click on the worksheet at the point where you want to start the calculation, then enter the calculation you desire - Example: Click somewhere in the space immediately below, then, type:  $5 + (1 + \sqrt{3 / 2 + \sin (p \langle \text{Cntl-G} \rangle / 3 =$

Enter your  
solution here ->

- \* The palettes on the right-hand side of the screen are used to enter mathematical items into your worksheet.
- \* To show/hide the palettes on the right-hand side of the screen, click on the very last icon in the control panel above. When you hover your mouse over this icon the legend "Show/hide side panel" shows up. The icon looks like this: 
- \* To enter special symbols, such as infinity, pi, i, absolute value, square root, etc., click on the corresponding symbol in the "Arithmetic" palette on the right-hand side of the screen.

Alternatively, use the following keystrokes (see Figure 1, below):

- |                               |                             |
|-------------------------------|-----------------------------|
| (1) Infinity:                 | Cntl+Shift+Z                |
| (2) Pi:                       | Cntl+Shift+P, or p <Cntl+G> |
| (3) i (imaginary unit):       | i                           |
| (4) Raise to a power:         | ^                           |
| (5) Absolute value:           | abs(                        |
| (6) Square root:              | \ or type sqrt(             |
| (7) N-th root:                | Cntl+\                      |
| (8) Symbolic evaluation (->): | Cntl+.                      |
| (9) Factorial:                | !                           |
| (10) Assignment to a variable | : (produces :=)             |
| (11) Numeric evaluation:      | =                           |

Other mathematical symbols (0-9 . + - \* / ,) can be entered directly from the keyboard.

NOTE: From here down, EXERCISES are shown to the right of the instructions or EXAMPLES given. To gain practice in the use of SMath Studio, solve the EXERCISES as shown below:

EXERCISES: Click on an empty space in the worksheet, then type the keystroke sequence shown between brackets:

- |     |                                 |   |
|-----|---------------------------------|---|
| 1 - | [1/<Cntl+Shift+Z><Cntl-.>]:     | : |
| 2 - | [sin(p <Cntl+G> /3 =]           | : |
| 3 - | [1/5+3*i=]                      | : |
| 4 - | [p <Cntl+G> ^ 2/3=]             | : |
| 5 - | [abs(2+3*i)=]                   | : |
| 6 - | [sqrt(-5=]                      | : |
| 7 - | [<Cntl+\>3 (top) 27 (bottom) =] | : |
| 8 - | [2.3^3 <Cntl+.>]                | : |
| 9 - | [20!/5!*15! =]                  | : |
| 10- | [a:sqrt(125)]                   | : |
| 11- | [a=]                            | : |

(12) The plus/minus operator produces a list of two values, as illustrated here:

$$\pm 3 = \begin{cases} 3 \\ -3 \end{cases}$$

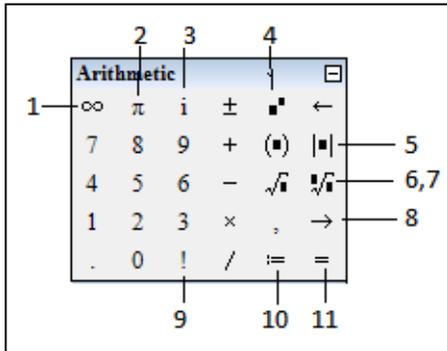


Figure 1. Mathematical symbols in the "Arithmetic" palette. See list above.

Any time that you end up with an operator that has placeholders (e.g., the square root symbol), you can just click on the placeholder to enter a value, or expression, in it. For example, the following images show the n-th root (Cntl+\), before and after the placeholders are filled with numbers:



$$\sqrt[3]{729} = 9$$

### 3 - Assigning values to variables

\* Variable names: a variable name in SMath Studio can be any collection of letters, numbers, or the undersign symbol `_`, as long as it starts with a letter. Some examples of variable names are:

`x`, `x1`, `x_1`, `x2y`, `xy`, `Xxy`, `X_23_y`, `Velocity_of_the_moon`, etc.

\* To define a variable: (1) click in an empty space in the worksheet, (2) type the name of the variable and use the assignment symbol (`:`) Then, (3) enter a value or expression to the right of the resulting assignment symbol (`:=`). (4) Click somewhere else in the worksheet.

The value to the right of the assignment symbol (`:=`) will now be assigned to the variable name to the left of it.

Some examples of variable assignments are shown below:

`x:=2`    `y1:=1.123`    `Radius_of_ball:=2+sqrt(13.5)`

EXERCISE: Click on an empty space in the worksheet, then type the keystroke sequence shown between brackets:

`[-3 <Click ±> sqrt(3^2 <space bar> -4*1*2 <space bar> <space bar> <space bar> <space bar> / 2 =]`

\* To see the value of a variable previously defined, type the variable name to the right of or below the variable definition, followed by the equal sign (=) or by the symbolic evaluation sign (->). Examples:

$$\begin{array}{l}
 x=2 \quad x=2 \qquad y1=1.123 \quad y1=\frac{1123}{1000} \\
 \\
 \text{Radius\_of\_ball}=5.6742 \quad \text{Radius\_of\_ball}=\frac{2^{\frac{3}{2}}+\sqrt{27}}{\sqrt{2}}
 \end{array}$$

\* Numerical vs. symbolic results:  
 =====

NOTE: Using = produces a numerical (floating-point) result, while using -> produces a symbolic result (ratio of integers, or expression involving radicals and fractions).

**EXERCISES:** Click on an empty space in the worksheet, then perform the required variable assignment. Afterwards, show the contents of the variable assigned by using either a numerical evaluation (=) or a symbolic evaluation (Cntl+.):

\* Unevaluated variables:  
 =====

If a variable has not been previously evaluated, it will show an empty placeholder in an assignment statement if we try to list that particular variable name. For example, at this point variable "Width" has not been evaluated yet, so, typing "Width = " [Enter] will produce:

Width:= █

<u>Variable name</u>	<u>Value to assign</u>
x01	38.2
u_A	sqrt(-23 <space bar>/3)
b	3.5
m	1.2
y	0.5

Thus, SMath Studio indicates that no value is yet assigned to that variable, and, by default, produces an assignment statement to evaluate that variable.

\* Previously-evaluated variables  
 =====

Variables evaluated earlier in a worksheet can be used to define other variables. For example, having evaluated variables x and y1 above, we can define the following variables:

$$\begin{array}{l}
 \text{Moment\_of\_inertia}=(x+y1)^2 \qquad \text{Moment\_of\_inertia}=9.7531 \\
 \\
 \text{Moment\_of\_inertia}=\frac{9753129}{1000000}
 \end{array}$$



An alternative way to enter Greek letters, that does not rely on the "Symbols" palettes, is to type the "equivalent" English letter, say, a, and immediately enter Cntl+G. Some examples are shown below. The typing sequence to the left indicates the typing sequence as well as the name of the Greek letter which is produced in the worksheet:

g Cntl+G (gamma):	$\gamma$	x Cntl+G (xi):	$\xi$
e Cntl+G (epsilon):	$\epsilon$	r Cntl+G (rho):	$\rho$
z Cntl+G (zeta):	$\zeta$	s Cntl+G (sigma):	$\sigma$
h Cntl+G (eta):	$\eta$	t Cntl+G (tau):	$\tau$
q Cntl+G (theta):	$\theta$	u Cntl+G (upsilon):	$\upsilon$
k Cntl+G (kappa):	$\kappa$	f Cntl+G (phi):	$\phi$
l Cntl+G (lambda):	$\lambda$	c Cntl+G (chi):	$\chi$
m Cntl+G (mu):	$\mu$	y Cntl+G (ypsilon):	$\psi$
n Cntl+G (nu):	$\nu$	w Cntl+G (omega):	$\omega$

EXERCISES: Click on an empty space in the worksheet, then perform the required variable assignment. Afterwards, show the contents of the variable assigned by using either a numerical evaluation (=) or a symbolic evaluation (Cntl+.):

<u>Variable name</u>	<u>Value to assign</u>
$\alpha$	38.2
$\phi\_A$	sqrt(123 <space bar>/3)
$\beta$	3.5
$\mu$	1.2
$\psi$	0.5
$\tau\_Top$	$\beta+2*\mu*\psi$
$E\_Area$	$(\beta+2*\mu*\psi)*\psi$
$\Omega\_Perim$	$\beta+2*\psi*\sqrt{1+\mu^2}$
$\eta\_Rad$	$E\_Area/\Omega\_Perim$

Typing an upper-case English letter followed by Cntl+G will produce the "equivalent" upper-case Greek character, e.g.,

G Cntl+G (Gamma):	$\Gamma$	S Cntl+G (Sigma):	$\Sigma$
Q Cntl+G (Theta):	$\Theta$	F Cntl+G (Phi):	$\Phi$
L Cntl+G (Lambda):	$\Lambda$	Y Cntl+G (Ypsilon):	$\Psi$
X Cntl+G (Xi):	$\Xi$	W Cntl+G (Omega):	$\Omega$

\* Using variables with subscripts:

=====+

Suppose you want to solve the following problem from physics:

**Curvilinear motion problem: [1]. Determine the speed  $v$  of a particle moving in a circular trajectory at time  $t = 20$  s along the trajectory, if the particle starts with a speed  $v_0 = 5.2$  m/s at time  $t_0 = 5$  m, and its tangential acceleration is  $a_t = 4.5$  m/s<sup>2</sup>. Use the equation:  $v = v_0 + a_t \cdot (t - t_0)$**

We can simply choose to not use subscripts, instead using two-character variable names for the subscripted variables, e.g.,

t:= 20      v0:= 5.2      t0:= 5      at:= 4.5

v:= v0+ at\*(t-t0)      thus,      v= 72.7

Alternatively, to make the solution look like it would in a textbook setting, we can use subscripted variables. To type the name of a subscripted variable, type first the name of the variable, then a period (.), then the subscript. In the following example, the characters between brackets show the typing sequence that produces the subscripted variables evaluation statement:

EXERCISE: Solve the following problem:

Curvilinear motion problem: [2]. Determine the normal acceleration,  $a_n$ , of a particle moving on a circular path of radius,  $\rho = 100$  m, if the speed of the particle is  $v = 20$  m/s. Use the equation:  $a_n = v^2/\rho$ . Do not include units in the calculations.

```
t:=20 [v.0:5.2] v_0:=5.2 [t.0:5] t_0:=5 [a.t:4.5] a_t:=4.5
[v:v.0+a.t*(t-t.0)] v:=v_0+a_t*(t-t_0) v=72.7
```

Notes:

- (1) We didn't include units in the definition of the variables above. The use of units will be introduced later.
- (2) The sequence <variable\_name>.<subscript> produces a subscripted variable only in mathematical expressions. This sequence does not work in text. Try: v.0 (this is in text). Therefore, it is not possible to type subscripted variables in text.
- (3) English and Greek letters can be used for variable names as well as for subscripts. Examples:

```
alpha_beta:=2.3 K_delta:=3.2 theta_k:=34.5
```

#### 4- Typing mathematical expressions

Some of the examples shown above used relatively simple mathematical expressions. Here we repeat some of them with the corresponding typing sequence between brackets:

```
[Radius_of_ball: 2 + sqrt 13.5]
```

```
Radius_of_ball=2+sqrt(13.5)
```

```
[Moment_of_inertia: (x + y1 <space bar> <space bar>) ^ 2]
```

```
Moment_of_inertia=(x+y1)^2
```

```
[d <Cntl+G> : a <Cntl+G> * ( a <Cntl+G> + b <Cntl+G>)]
```

```
delta:=alpha*(alpha+beta)
```

```
[v:v.0+a.t*(t-t.0)]
```

```
v:=v_0+a_t*(t-t_0)
```

Techniques for building mathematical expressions:

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The examples above include some particular character sequences, namely:

sqrt	to enter the square root symbol
<Cntl+G>	to enter Greek letters
<space bar>	to select wider expressions or expressions between parentheses

**EXERCISE:** Solve the following problem:

Thermal expansion problem [1]. Determine the thermal expansion  $\Delta L$  suffered by a rod of length  $L.0 = 10$  cm if the initial temperature is  $T.0 = 20$  centigrade, the final temperature is  $T.f = 40$  centigrade, and the coefficient of thermal expansion is  $\alpha.S = 0.005$  cm/centigrade. Also, determine the final length,  $L.f$ , of the rod after the expansion. Use the equations:

$$\Delta L = \alpha.S*(T.f-T.0), \text{ and } L.f = L.0 + \Delta L$$

Do not include units in the calculations.

Another important typing sequence is the use of the right-arrow key in your keyboard. It's action is similar to that of the space bar, namely, to select wider expressions or expressions between parentheses.

Two-dimensional (2D) mathematical expressions:

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The mathematical expressions produced by SMath Studio are referred to as two-dimensional (2D) math, namely, exponents, fractions, etc., will be shown in their proper place as in textbook style. Here's another example:

[t <Cntl+G> : t <Cntl+G>.0+a <Cntl+G>.c \* (T.f-T.0 <space bar> <space bar>  
/ sqrt(1+ abs( D <Cntl+G> T /b <Cntl+G> <space bar> <space bar> ^3 <space bar>  
<space bar> <space bar> <space bar> ^ 3/2]

$$\tau := \tau_0 + \alpha_c \cdot \left( \frac{(T_f - T_0)}{\sqrt{1 + \left| \frac{\Delta T}{\beta} \right|^3}} \right)^{\frac{3}{2}} \quad \tau = \blacksquare$$

This example shows the use of Greek letters (Cntl+G), parentheses, space bar, subscripted variables, sqrt (square root), and abs (absolute value) in a 2D mathematical expressions. With practice you will learn to build complex 2D mathematics in SMath Studio.

Correcting errors:

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Correcting errors in entering an expression can be accomplished in many ways:

- 1 - Use the Backspace key to remove the last key stroke(s).
- 2 - Select elements to be removed using your mouse, then the Backspace key.
- 3 - Click on a specific location to insert variables or expressions.
- 4 - You can copy and paste an expression within another expression.

Using the Boolean equal sign for non-evaluating expressions:

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The example above is an assignment statement. The variables in the right-hand side of the assignment statement should be available beforehand in order for this statement to produce a result. In this example those variables are undefined. If you click on the expression  $\tau =$  to the right the message " $\tau.0$  - not defined" will be shown.

Sometimes you need to type an expression not with the purpose of evaluating it, but with the purpose of documenting an assignment or report. In such case, you can use the Boolean equal sign (first sign in the "Boolean" palette in the right-hand side panel in the SMath Studio interface, also entered by using Cntl+=) to build your non-evaluating mathematical expression. Thus, you could type, for example:

```
[t <Cntl+G> <Cntl+=> t <Cntl+G>.0+a <Cntl+G>.c * (T.f-T.0 <space bar> <space bar>
 / sqrt(1+ abs( D <Cntl+G> T /b <Cntl+G> <space bar> <space bar> ^3 <space bar>
 <space bar> <space bar> <space bar> ^ 3/2]
```

$$\tau = \tau_0 + \alpha_c \cdot \left( \frac{(T_f - T_0)}{\sqrt{1 + \left| \frac{\Delta T}{\beta} \right|^3}} \right)^{\frac{3}{2}}$$

The result is simply a mathematical expression that includes no evaluations.

Using the Boolean equal sign with function "solve":

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One other use of the Boolean equal sign is to build a mathematical equation to be solved using function "solve". For example, the quadratic equation:

$$a \cdot x^2 + b \cdot x + c = 0 \quad \text{has the well known solution:} \quad x = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a}$$

Thus, if we take: a:=2 b:=5 c:=-6 we calculate solutions as follows:

$$x = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a}, \quad \text{i.e.,} \quad x = \begin{cases} 0.886 \\ -3.386 \end{cases}$$

Note: to highlight a solution you could place a frame around it by using the "Border on/off" button in the menu bar (it looks like a square frame). First, you need to select the solution field by dragging your mouse while holding down the left key over it.

You can also highlight the framed solution by using a background color as follows: (1) Drag your mouse while holding the left key over the solution field; (2) Click on the "Background color" button in the menu bar (it looks like a painter's palette); (3) Click on one of the color squares; (4) Press the [ OK ] button. Here's an example of highlighting a solution with both a frame and a background color:

$$x = \begin{cases} 0.886 \\ -3.386 \end{cases}$$

An alternative way to solve the quadratic equation used above, namely,

$$2 \cdot x^2 + 5 \cdot x - 6 = 0$$

EXERCISE: Type the following expression in SMath Studio, as a non-evaluating expression (i.e., use a Boolean =)

$$Q = \frac{C_u}{n} \cdot \frac{((b+m \cdot y) \cdot y)^{\frac{5}{3}} \cdot \sqrt{S_0}}{(b+2 \cdot y \cdot \sqrt{1+m^2})^{\frac{2}{3}}}$$

EXERCISES: Use the quadratic equation formula to solve the following equations:

$$-3 \cdot y^2 + 2 \cdot y + 6 = 0$$

$$2.5 \cdot \xi^2 - 7.5 \cdot \xi + 16.5 = 0$$

$$\lambda^2 - 3 \cdot \lambda + 25 = 0$$

$$25 \cdot \beta^2 - 76 = 0$$

is to use function "solve". Here we show two different ways to call function "solve":

1 - Using function "solve" with two arguments:

$$\text{solve}(2 \cdot x^2 + 5 \cdot x - 6 = 0, x) = \begin{pmatrix} -3.386 \\ 0.886 \end{pmatrix}$$

2 - Using function "solve" with four arguments, limiting the range of the solution:

$$\text{solve}(2 \cdot x^2 + 5 \cdot x - 6 = 0, x, 0, 10) = 0.886$$

An alternative way to use function "solve" is to assign the function call to a variable, e.g.,

$$x := \text{solve}(2 \cdot x^2 + 5 \cdot x - 6 = 0, x) \quad , \text{ whose solution is: } \quad x = \begin{pmatrix} -3.386 \\ 0.886 \end{pmatrix}$$

The "solve" function call can be assigned to another variable, e.g.,

$$\xi := \text{solve}(2 \cdot x^2 + 5 \cdot x - 6 = 0, x) \quad , \text{ whose solution is: } \quad \xi = \begin{pmatrix} -3.386 \\ 0.886 \end{pmatrix}$$

Other functions used in mathematical expressions:

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The "Functions" palette in the right-hand side of the SMath Studio interface shows a number of mathematical functions that can be used in building mathematical expressions: log (logarithm of any base), sign, sin, cos, ln (natural log), arg (angle for a complex number), tan, cot, exp, and % (percentage).

To type any of these functions in an expression or to evaluate them, simply click on the corresponding function name in the "Functions" palette. All these functions can also be typed, for example:

```
[log(10,2):]   log2(10)=3.3219    , i.e., log 10 of base 2
              sign(-10)=-1
              sin(1.2)+cos(1.2)=1.2944
              arg(4.5-3.2*i)=-0.6181
              ln(10)=2.3026
              exp(-0.2)=0.8187
              perc(20,5)=1        , i.e., 20% of 5
```

EXERCISES: Use function "solve" to solve the following equations:

$$-3 \cdot y^2 + 2 \cdot y + 6 = 0$$

$$2.5 \cdot \xi^2 - 7.5 \cdot \xi + 16.5 = 0$$

$$\lambda^2 - 3 \cdot \lambda + 25 = 0$$

$$25 \cdot \beta^2 - 76 = 0$$

$$3 \cdot t^3 - 2 \cdot t + 5 = 0$$

$$3 \cdot t^4 - 2 \cdot t^2 - 5 = 0$$

$$5 \cdot \eta^4 - 5 \cdot \eta - 5 = 0$$

EXERCISES: Calculate the following:

- 1 - log base 5 of 625 :
- 2 - sign of -2.5 :
- 3 - sine of 1.75 rad :
- 4 - cosine of 0.85 rad :
- 5 - natural log of 125 :
- 6 - natural log of -124:
- 7 - ln(1+3\*i) :
- 8 - exp(2.35) :
- 9 - exp(1-i) :
- 10- 5% of 120 :

Notes: (1) The letters  $i$  (imaginary unit),  $\pi$  (ratio of circumference to diameter in a circle), and  $e$  (base of natural logarithms), are predefined constants and can be used directly in mathematical expressions. Thus, the last statement above, namely " $\text{exp}(-0.2) = 0.8187$ " can also be written as:

$$e^{-0.2} = 0.8187$$

(2) By default, the arguments of trigonometric functions ( $\sin$ ,  $\cos$ ,  $\tan$ ,  $\cot$ ) must be given in radians, e.g.,

$$\sin\left(\frac{\pi}{3}\right) = 0.866 \qquad \cos\left(\frac{\pi}{4}\right) = 0.7071$$

If you want to use degrees in your arguments of trigonometric functions, type the number of degrees followed by 'deg (the quote must be included), e.g.,

$$\begin{aligned} [\sin(60\text{'deg})=] & \qquad [\cos(45\text{'deg})=] \\ \sin(60\text{ deg}) = 0.866 & \qquad \cos(45\text{ deg}) = 0.7071 \end{aligned}$$

**EXERCISES:** Calculate the following:

- 1 -  $\ln(e^{-2.5})$  :
- 2 -  $\sin(1.5)^2 + \cos(1.5)^2$  :
- 3 -  $\sin(28.5 \text{ deg})$  :
- 4 -  $\tan(123.6 \text{ deg})$  :

**EXERCISES:** Use function "solve" to solve the following equations:

- $\ln(r) - 1.5 = 0$
- $e^{-x} = 0.25$
- $e^{2 \cdot \xi} - e^{\xi} = 3$
- $e^{\frac{r}{2}} - \ln\left(\frac{r}{2}\right) = 3$
- $\sin(r) - \cos(r) = 0.25 \quad \text{in } 0 < r < 2\pi$
- $\frac{\sin(\phi)}{\phi} - 0.1 = 0 \quad \text{in } -\pi < \phi < \pi$

Additional functions in the "Functions" palette:

Besides the 10 functions listed above, the "Functions" palette also includes the following 8 functions (see Figure 3, below):

- 1 - summation      2 - product      3 - derivative      4 - definite integral
- 5 - element of a vector      6 - multiple values
- 7 - two-dimensional (2D) graph      8 - three-dimensional (3D) graph

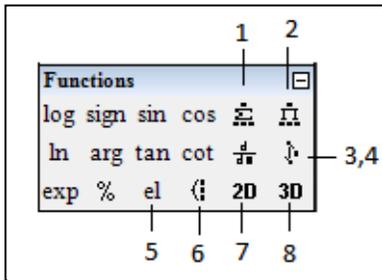


Figure 3. Details of the "Functions" palette.

Symbols (6), (7) and (8) apply to the production of graphs. These three symbols will be discussed in another section. Here, we concentrate on describing symbols (1) through (5) in the "Functions" palette.

To enter a summation, product, derivative, or definite integral symbol, simply select the location of the entry point in the worksheet and click the corresponding symbol. Symbols (1), (2) and (4) will show four placeholders, while symbol (3) will show two placeholders. Summations, products, and integrals can be calculated using a numerical evaluation (=) or the symbolic evaluation (Cntl+.). Derivatives can only be evaluated using symbolic evaluation (Cntl+.).

Here are some examples:

Symbol 1, summation, as first clicked - Example:

$$\sum_{k=1}^{10} \left(\frac{1}{k^2}\right) = 1.5498 \quad \sum_{k=1}^{10} \left(\frac{1}{k^2}\right) = \frac{1968329}{1270080}$$

$$S_1 = \frac{1}{n} \cdot \sum_{j=1}^n \left(\frac{3}{j}\right)^3, \text{ for } n = 5$$

Symbol 2, product, as first clicked - Example:

$$\prod_{r=5}^8 \sqrt{(r-2)} = 18.9737 \quad \prod_{r=5}^8 \sqrt{(r-2)} = \sqrt{3} \cdot \sqrt{4} \cdot \sqrt{5} \cdot \sqrt{6}$$

$$x_g = \left( \prod_{k=1}^m \left(\frac{k}{10}\right)^2 \right)^{1/m}, \text{ for } m = 10$$

Symbol 3, derivative, as first clicked - Examples:

$$\frac{d}{d\eta} (\eta^2 + \ln(\eta)) = \frac{1 + 2 \cdot \eta}{\eta} \quad \frac{d}{d\zeta} \tan(\zeta) = \frac{1}{\cos(\zeta)^2}$$

Symbol 4, integral, as first clicked Example:

$$\int_0^3 \frac{1}{1+s^2} ds = 1.249 \quad \int_0^3 \frac{1}{1+s^2} ds = \frac{631970953161275}{505963005627923}$$

$$\frac{d}{dr} (r \cdot \ln(r) - r^2)$$

Symbol 5 produces two placeholders, a main placeholder to contain the name of a vector and a subindex placeholder to contain a numerical value representing the order of an element in the vector. We present two examples here:

1 - Define, for example, the 5-th element of a vector w using "el", so as to create a new vector of 5 elements:

```
[el(w,5 <space bar> :-4.5)      w_5 := -4.5
```

$$y_c = \frac{\int_0^h y \cdot \sin(y) \cdot (3 + 2 \cdot y) dy}{\int_0^h \cos(y) \cdot (3 + 2 \cdot y^2) dy}, \text{ for } h = 2$$

If we now list w, we get a column vector of 5 elements with all zero elements, but the last one:

$$w = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -4.5 \end{pmatrix}$$

If you now want to enter other values in the zero elements, use the "el" function with indices 1 through 4, e.g., replacing elements 1 and 3:

$$w_1 := 4.2$$

$$w_3 := -6.2$$

The result is now:

$$w = \begin{pmatrix} 4.2 \\ 0 \\ -6.2 \\ -4.5 \end{pmatrix}$$

2 - Define, for example, the element in the third row and fourth column of a matrix R using "el", so as to create a 3x4 matrix:

$$[\text{el}(R, 3, 4 \text{ <space bar> } : 4.3 \quad R_{3 \ 4} := 4.3$$

Listing R we get the required 3x4 matrix with all elements, but the very last one, as zero:

$$R = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4.3 \end{pmatrix}$$

If you now want to enter other values in the zero elements, use the "el" function with other indices within the 3x4 matrix, e.g.,

$$R_{1 \ 1} := -2$$

$$R_{2 \ 3} := 3.1$$

$$R_{3 \ 2} := -7$$

The result is now:

$$R = \begin{pmatrix} -2 & 0 & 0 & 0 \\ 0 & 0 & 3.1 & 0 \\ 0 & -7 & 0 & 4.3 \end{pmatrix}$$

**EXERCISES:** Use function "el" to create the following vectors and matrices:

$$uA = \begin{pmatrix} -2 \\ 0 \\ 1 \\ 4 \end{pmatrix} \quad uB = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix} \quad AC = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix} \quad BC = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

Notes: (1) Instead of function "el" you can simply type the name of the vector (or matrix), then type [ and enter the largest subindex (or subindices for a matrix), followed by <space bar> and an assignment.

Examples:

$$[s[4 \text{ <space bar> } : 5] \quad s_4 := 5 \quad \text{to produce} \quad s = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 5 \end{pmatrix}$$

Other values can be entered by a similar approach, e.g.,

```
[s[2 <space bar> : 3]    s_2:=3
[s[1 <space bar> : -2]   s_1:=-2
```

The vector  $s$  is now:

$$s = \begin{pmatrix} -2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$$

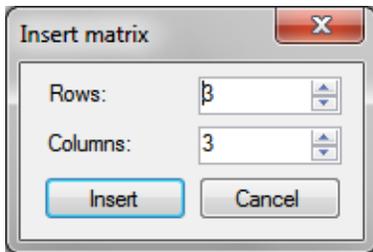
(2) Variable names with subindices, created by either using "el" or by using brackets ([), are different than variable names with subscripts, created by using the period (.). Example:

```
- subscript - use (.): [m.3:4]    m_3:=4    and    m:=■    m not defined
```

```
- subindex - use ([): [m[3 <space bar> :4]    m_3:=4    and    m = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix}
```

In this example, even though, both the expressions for  $m_3$ , with subscript and subindex, look the same, they produce different mathematical objects.

(3) A better way to build a vector or matrix, rather than using function "el", is by using the "Matrix" button (first button) in the "Matrices" palette, or, equivalently, typing "Cntl+m". This produces an entry form as illustrated below:



By default, the entry form shows 3 Rows and 3 Columns. You can change these values to any positive number. Remember:

- column vector: Rows= n    Columns= 1
- row vector:    Rows= 1    Columns= m
- nxm matrix:    Rows= n    Columns= m

```
As an example, use: [ v: <Cntl+m> Rows: 3 Columns:1 [Insert]]
```

This produces:  $v := \begin{pmatrix} \blacksquare \\ \blacksquare \\ \blacksquare \end{pmatrix}$  Next, click on each placeholder and enter the individual values of the element. When done, click outside of the vector.

**EXERCISES:** Use brackets to create the following vectors and matrices:

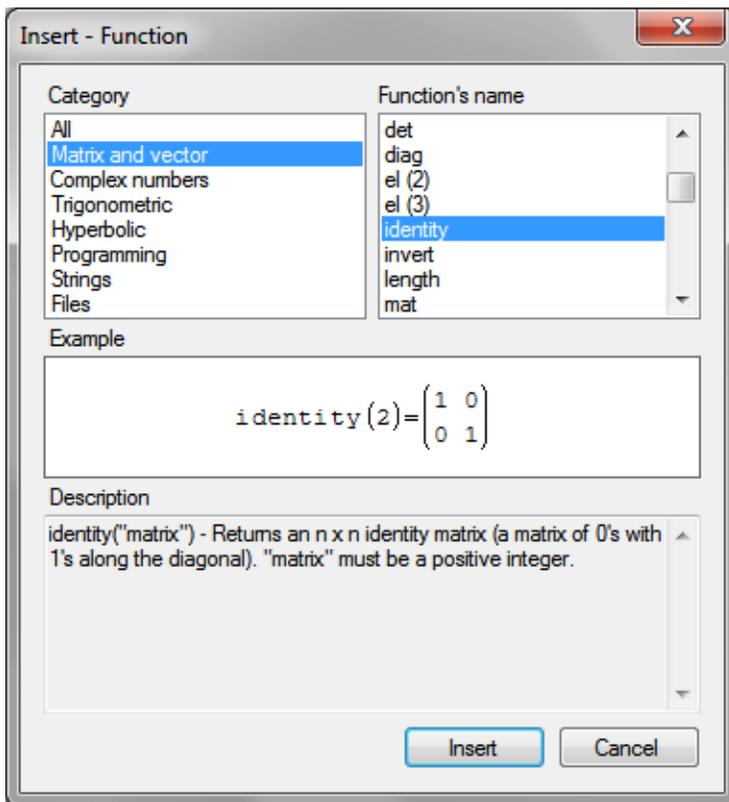
$$vA = \begin{pmatrix} 1 \\ -2 \\ 3 \\ 5 \end{pmatrix} \quad vB = \begin{pmatrix} 5 \\ 4 \\ 3 \\ 2 \\ 1 \end{pmatrix} \quad Ec = \begin{pmatrix} 6 & 5 \\ 4 & 3 \\ 2 & 1 \end{pmatrix} \quad Fc = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

For example, for this case we can use:  $v := \begin{pmatrix} 1.2 \\ -6.5 \\ 8.7 \end{pmatrix}$

(4) Detailed explanation of the "Matrices" palette, and operations with matrices will be provided later.

Functions from the "Function" menu:

Click on the button [f(x)] in the menu bar to generate a "Function" menu. The menu looks as shown in the figure below:



The "All" category will list all functions available in SMath Studio. Click in any other category to see a listing of functions specific to that category. Click on a specific function and an "Example" and a "Description" of the selected function will be displayed. Follow the Example and Description to use the function.

To insert the function, select the insertion point, then select the function name. Then, click the [Insert] button. Here's an example of inserting function "identity":

$$\text{identity}(4) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Alternatively, you can type the name of the function at the insertion point, and enter the corresponding arguments to use the function.

For the purpose of writing mathematical expressions, I suggest, at this point, to stick to the following functions categories: Complex number, Trigonometric, Hyperbolic. Also, use those in the "Arithmetic" and "Functions" palettes in the right-hand side panels of the interface. Don't forget, also, to use the Boolean equal sign (Cntrl+=) to write unevaluating expressions and for use with the "solve" function in solving equations.

**EXERCISE:** Use the "Matrix" button to create the following vectors and matrices:

$$wA = \begin{pmatrix} 0 \\ -1 \\ 0 \\ -5 \end{pmatrix} \quad wB = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 2 \\ 1 \end{pmatrix} \quad Hc = \begin{pmatrix} 6 & 5 \\ 5 & 6 \\ 6 & 5 \end{pmatrix}$$

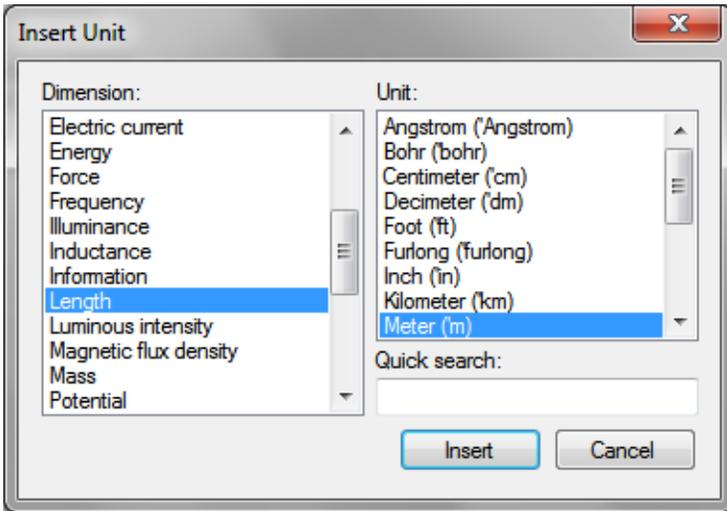
$$Kc = \begin{pmatrix} 1 & -1 & 1 \\ -2 & 1 & -3 \end{pmatrix}$$

**EXERCISES:** Calculate the following hyperbolic and inverse functions:

$$\begin{array}{ll} 1 - \sinh(2.3) & : \\ 2 - \cosh(1.2) & : \\ 3 - \operatorname{asinh}(2.4) & : \\ 4 - \operatorname{acosh}(0.5) & : \end{array}$$

### 5 - Using units

The "Unit" menu in the menu bar is the blue icon that looks like a funnel located immediately to the right of the "Function" menu [f(x)]. To insert units, type the numerical value accompanying the units, then click on the "Unit" menu, and select the proper unit to insert. The "Unit" menu looks as follows:



Select the "Dimension" in the left-hand-side list, and the "Unit" from the right-hand-side list. In this figure I have selected the "Length" dimension and the "Meter ('m)" unit. If I wanted to define a variable, say  $\lambda = 0.3 \text{ m}$ , I would use:

[1 <Cntrl+G> : 0.3 <Unit menu, select Meter> press [Insert]]

$$\lambda := 0.3 \text{ m}$$

Alternatively, simply type:

[1 <Cntrl+G> : 0.3 'm]:  $\lambda := 0.3 \text{ m}$

Thus, if you know the symbol for the unit to enter you only need to type ' followed by the unit symbol after the numerical value.

For example, we repeat the solution of this physics problems, but now using units:

**Curvilinear motion problem: [1]. Determine the speed  $v$  of a particle moving in a circular trajectory at time  $t = 20 \text{ s}$  along the trajectory, if the particle starts with a speed  $v_0 = 5.2 \text{ m/s}$  at time  $t_0 = 5 \text{ m}$ , and its tangential acceleration is  $a_t = 4.5 \text{ m/s}^2$ . Use the equation:**  $v = v_0 + a_t \cdot (t - t_0)$

[t:20's]     t := 20 s     [v.0:5.2'm/'s]      $v_0 := 5.2 \frac{\text{m}}{\text{s}}$

[t.0:5's]      $t_0 := 5 \text{ s}$      [a.t:4.5'm/'s^2]      $a_t := 4.5 \frac{\text{m}}{\text{s}^2}$

Solution:

[v:v.0+a.t\*(t-t.0)]      $v := v_0 + a_t \cdot (t - t_0)$       $v = 72.7 \frac{\text{m}}{\text{s}}$

**EXERCISE:** Calculate the following inverse trigonometric functions:

- 1 -  $\text{asin}(0.8)$      :
- 2 -  $\text{acos}(0.6)$      :
- 3 -  $\text{atan}(3.5)$      :

**EXERCISES:** For the following complex numbers:

$z_1 = 3+2*i$      $z_2 = -4+6*i$   
 $z_3 = 5-8*i$      $z_4 = \pi + \text{sqrt}(2)*i$

Calculate the following:

- 1 -  $\text{Re}(z_1)$ , real part :
- 2 -  $\text{Im}(z_2)$ , imag. part:
- 3 -  $\text{abs}(z_3)$ , modulus :
- 4 -  $\text{arg}(z_4)$ , angle     :

**EXERCISES:** Use functions "pol2xy" and "xy2pol" to convert from polar to Cartesian coordinates, or from Cartesian to polar coords. These functions are available in [f(x)] under "Complex number":

1 - Convert  $r = 3, \theta = \pi/3$  to (x,y):

2 - Convert  $x = -5, y = 2$ , to (r,θ)

**EXERCISE:** Solve the following problem: Curvilinear motion problem: [2]. Determine the normal acceleration,  $a_n$ , of a particle moving on a circular path of radius,  $\rho = 100 \text{ m}$ , if the speed of the particle is  $v = 20 \text{ m/s}$ . Use the equation:  $a_n = v^2/\rho$ . Calculate using units.

Notes: (1) Check the list of units in the "Unit" menu to make sure that the particular unit you are seeking is defined. For example, in the units of "Length" you find km, dm, m, cm, mm,  $\mu\text{m}$ , nm, pm. So, if you needed to use, say, a hm (hectameter = 100 m), such a unit will not be available.

(2) You can define your own units. For example, you can define the hectameter as follows:

`['hm:100'm]      hm:= 100 m`

In Fluid Mechanics/Hydraulics, we could define a unit of flow velocity, the fps (feet per second), or a unit of discharge, the gpm (gallons per minute), as follows:

`['fps:'ft/'s]      fps:=  $\frac{ft}{s}$       ['gpm:'gal/'min]      gpm:=  $\frac{gal}{min}$`

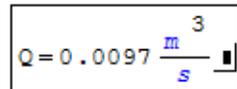
(3) The result of calculations involving units will default to units of the SI (International System), for example:

Define: `[D:5'in]`      `D:= 5 in`      Check:      `D=0.127 m`

Define: `[V:= 2.5'fps]`      `V:= 2.5 fps`      Check:      `V=0.762  $\frac{m}{s}$`

Calculate: `[Q:=V* $\pi$ *D^2/4]`      `Q:=V* $\pi$ * $\frac{D^2}{4}$`       Result:      `Q=0.0097  $\frac{m^3}{s}$`

(4) To convert your result to, say, gpm, click in the field containing the result. You will see a placeholder to the right of the result (see figure below -- this is only a figure, not a field where you can make any changes):



$Q = 0.0097 \frac{m^3}{s}$  ■

Click on the placeholder, and type the units you want to replace, e.g., 'gpm (using the 'gpm units defined earlier), to get:

`Q=152.9996 gpm`

(5) You cannot use units in any term in an equation to be solved by the "solve" command. For example, to solve the quadratic equation with:

$$a := 2 \frac{s}{m} \quad b := 5 \frac{s}{m} \quad c := -6$$

**EXERCISE:** Solve the following problem: Thermal expansion problem [1]. Determine the thermal expansion  $\Delta L$  suffered by a rod of length  $L_0 = 10$  cm if the initial temperature is  $T_0 = 20$  centigrade, the final temperature is  $T_f = 40$  centigrade, and the coefficient of thermal expansion is  $\alpha_S = 0.005$  cm/centigrade. Also, determine the final length,  $L_f$ , of the rod after the expansion. Use the equations:

$$\Delta L = \alpha_S (T_f - T_0), \text{ and } L_f = L_0 + \Delta L$$

Calculate using units.

We can use the formula:

$$x := \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a}, \text{ i.e., } x = \begin{cases} 0.886 \frac{m}{s} \\ -3.386 \frac{m}{s} \end{cases}$$

Since this result was obtained through a simple algebraic calculation, the results are showing with the proper units. However, an attempt to solve the equation using "solve" will fail:

$$\text{solve}(a \cdot x^2 + b \cdot x + c = 0, x) = \blacksquare$$

If you click on the equation above, you'll get the error message: "No real roots".

Thus, you can only use "solve" to find the solution to an equation not involving units. For the case of interest, shown above, we could try the following:

$$\text{solve}\left(\frac{a}{\frac{s}{m}^2} \cdot x^2 + \frac{b}{\frac{s}{m}} \cdot x + c = 0, x\right) = \begin{pmatrix} -3.386 \\ 0.886 \end{pmatrix} \quad \text{Result in 'm/s'}$$

By dividing a by 's^2/m^2' and b by 's/m' we are effectively eliminating the units from the equation, and allowing "solve" to deal with a unitless expression. The results shown are in 'm/s', as expected.

Let's check what happens if we use values of a and b in units of the English System -- Here I'm using the equivalent values:

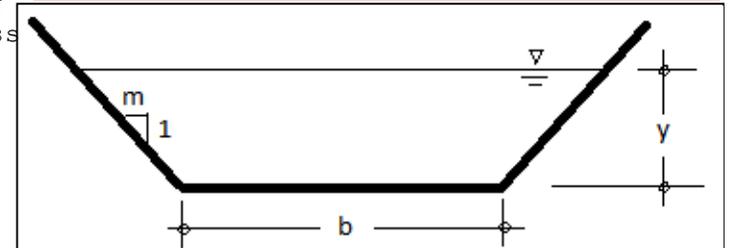
$$2 \frac{s}{m}^2 = 0.1858 \frac{s}{ft}^2 \quad 5 \frac{s}{m} = 1.524 \frac{s}{ft}$$

$$\text{i.e., } a = 0.1858 \frac{s}{ft}^2 \quad b = 1.524 \frac{s}{ft}$$

Now, I'll use the original "solve" function called shown above, namely:

$$\text{solve}\left(\frac{a}{\frac{s}{m}^2} \cdot x^2 + \frac{b}{\frac{s}{m}} \cdot x + c = 0, x\right) = \begin{pmatrix} -3.3861 \\ 0.886 \end{pmatrix} \quad \text{With result in 'm/s'}$$

**EXERCISE: Solve the following problem:  
Open Channel Flow in a Trapezoidal Channel.**



NOTE: Before solving this problem, define the following unit: 'cfs' = 'ft^3/s', i.e., cfs = cubic feet per second.

[1]. A Trapezoidal channel with a bottom width,  $b = 5$  ft, and a side slope,  $m = 1/2$ , is laid on a slope,  $S.0 = 0.00023$ , and lined with concrete (Manning's  $n = 0.012$ ). For a discharge  $Q = 20$  cfs, determine the depth of flow,  $y$ , using Manning's equation:

We can also use:  $\text{solve}\left(\frac{a}{\frac{s}{ft}^2} \cdot x^2 + \frac{b}{\frac{s}{ft}} \cdot x + c = 0, x\right) = \begin{pmatrix} -11.1092 \\ 2.9068 \end{pmatrix}$  With result in 'ft/'s

$$Q = \frac{C_u}{n} \cdot \frac{((b + m \cdot y) \cdot y)^{\frac{5}{3}}}{(b + 2 \cdot y \cdot \sqrt{1 + m^2})^{\frac{2}{3}}} \cdot \sqrt{S_0}$$

Let's check:  $-11.1092 \frac{ft}{s} = -3.3861 \frac{m}{s}$ , and  $2.9068 \frac{ft}{s} = 0.886 \frac{m}{s}$

When using "solve," therefore, my advice is to use dimensionally consistent equations, without units, but making sure to track the proper units in the result. Consider the following application:

In units of the English System, the constant C.u is C.u = 1.486 ft^(1/3)/s.

$$t := 20 \text{ s} \quad v := 72.7 \frac{m}{s} \quad t_0 := 5 \text{ s} \quad a_t := 4.5 \frac{m}{s^2}$$

To solve this problem use the "solve" function, with Manning's equation (see above), but eliminating the units by re-writing the equation as follows:

$$\text{solve}\left(\frac{v}{\frac{m}{s}} = v_0 + \frac{a_t}{\frac{m}{s^2}} \cdot \left(\frac{t}{s} - \frac{t_0}{s}\right), v_0\right) = 5.2$$

$$\frac{Q}{'cfs} = \frac{\frac{C_u}{'ft^{\frac{1}{3}}}}{n} \cdot \frac{((\frac{b}{'ft} + m \cdot y) \cdot y)^{\frac{5}{3}}}{(\frac{b}{'ft} + 2 \cdot y \cdot \sqrt{1 + m^2})^{\frac{2}{3}}} \cdot \sqrt{S_0}$$

The variable we are solving for, namely, v.0, is not accompanied by units. The results should be in the proper units of 'm/'s.

Alternatively, you could define the values of the known variables as pure numbers, with the labels for the units typed in text fields next to the numerical values, e.g.,

Solve for y, the result should be in ft.

$$t := 20 \text{ s} \quad v := 72.7 \text{ m/s} \quad t_0 := 5 \text{ s} \quad a_t := 4.5 \text{ m/s}^2$$

Then, use:  $\text{solve}(v = v_0 + a_t \cdot (t - t_0), v_0) = 5.2 \text{ m/s}$

## 6 - Defining your own functions

Above we showed how to use the functions in the "Functions" palette as well as in the "Function" menu [f(x)]. These functions, which are readily available in SMATH Studio for your use, are referred to as "predefined functions." Functions defined by the user are, of course, "user-defined functions."

A user-defined function can be defined by using the general format:

<function\_name>( <argument(s)> ) = <expression involving arguments(s)>

Example:

$$f_0(x) := \sqrt{x^2 + 2} \qquad f_0(2.5) = \frac{\sqrt{33}}{\sqrt{4}}$$

\* You can evaluate the function at a number:  $f_0(2.4) = 2.7857$

\* You can evaluate the function at a variable previously defined:

$$\xi := -1.34 \quad f_0(\xi) = 1.9482 \quad f_0(\xi) = \frac{\sqrt{9489}}{\sqrt{2500}}$$

\* You can use the function in an equation, e.g.,:

$$\text{solve}(f_0(t) - 2.5 = 0, t) = \begin{pmatrix} -2.0616 \\ 2.0616 \end{pmatrix}$$

You can define a multivariate function (i.e., a function of more than one variable), for example:

$$f_1(x, y) := x^2 + y^2 - 25$$

Evaluations of  $f_1(x, y)$  are shown next:

$$f_1(2, 6) = 15 \quad f_1(\sqrt{2}, \sqrt{3}) = -20 \quad \alpha := 2 \quad \beta := 5 \quad f_1(\alpha, \beta) = 4$$

You can use the function in an equation, e.g.,

$$\text{solve}(f_1(2, y) = 0, y) = \begin{pmatrix} -4.5826 \\ 4.5826 \end{pmatrix}$$

EXAMPLE - Defining the pmf and CDF for the Binomial distribution:

=====

For a discrete random variable  $X$  that follows the Binomial distribution with parameters  $n$  (positive integer) and  $p$  ( $0 < p < 1$ ), the probability mass function (pmf),  $f(x)$ , is given by:

$$f(x) := \frac{n!}{(x!) \cdot ((n-x)!)} \cdot p^x \cdot (1-p)^{n-x}, \text{ for } x = 0, 1, \dots, n$$

The cumulative distribution function (CDF),  $F(x)$ , is, in turn, defined as:

$$F(x) := \sum_{k=0}^x f(k), \text{ for } x = 0, 1, \dots, n$$

EXERCISES: Define a function

$$h(y) = 1/(1+y^2), \text{ then}$$

- (1) Evaluate  $h(2)$
- (2) Evaluate  $h(\pi/2)$
- (3) Evaluate  $h(3.5) + h(1.2)^2$
- (4) Solve the equation:  $h(y) = 0.05$  using function "solve"
- (5) Determine an expression for the derivative of  $h(y)$  with respect to  $y$ . Use symbolic evaluation.
- (6) Calculate the summation of  $\text{sqrt}(h(k))$  for  $k$  from 1 to 5.
- (7) Calculate the definite integral of  $h(\xi)^2$  from  $\xi = 1$  to 5

For particular values:  $n:=10$      $p:=0.30$     we can calculate some values of the pdf and CDF:

$$\begin{array}{llll} f(3)=0.2668 & f(5)=0.1029 & f(7)=0.009 & f(9)=0.0001 \\ F(3)=0.6496 & F(5)=0.9527 & F(7)=0.9984 & F(9)=1 \end{array}$$

EXERCISE - For the Binomial variable used in the EXAMPLE, with  $n = 10$ ,  $p = 0.3$ , calculate the following probability:

$$P(3 \leq X \leq 6) = \sum_{k=3}^6 f(k)$$

EXAMPLE - Defining the pdf and CDF for the Exponential distribution:

=====

For a continuous random variable  $X$  that follows the exponential distribution with parameter  $\mu$  (a positive real number), the probability density function (pdf),  $f(x)$ , is given by:

$$f(x) := \mu \cdot e^{-\mu \cdot x}, \text{ for } x \geq 0$$

Compare the last result with that given by:

$$P(3 \leq X \leq 6) = F(6) - F(2)$$

NOTE: Typically,  $\lambda$  is used as the parameter for the exponential distribution. In this worksheet I have already used variable  $\lambda$ , which currently holds the value:

$$\lambda = 0.3 \text{ m}$$

Thus, to prevent conflicts with the variable  $\lambda$  in defining the pdf above, I decided instead to use the variable  $\mu$  as the exponential distribution parameter.

For the following value:  $\mu := 4.5$  we can calculate some values of the pdf as follows:

$$f(0.1) = 2.8693 \quad f(0.5) = 0.4743 \quad f(1.0) = 0.05 \quad f(1.5) = 0.0053$$

The cumulative distribution function (CDF),  $F(x)$ , can be defined as:

$$F(x) := \int_0^x f(t) dt, \text{ for } x = 0, 1, \dots, n$$

However, attempts to calculate  $F(x)$  for selected values of  $x$  fail:

$$F(0.1) = \blacksquare \quad F(0.5) = \blacksquare \quad F(1.0) = \blacksquare \quad F(1.5) = \blacksquare$$

EXERCISE - For the Exponential variable used in the EXAMPLE, with  $\mu = 4.5$ , calculate the following probability:

$$P(0.2 < X < 1.2) = \int_{0.2}^{1.2} f(\xi) d\xi$$

NOTE: The reason why this definition of  $F(x)$  fails to produce values is that I have used already variables  $x$ , and  $t$ . Thus, these variables already have these values associated with them:

$$x = \begin{cases} 0.886 \frac{m}{s} \\ -3.386 \frac{m}{s} \end{cases} \quad t = 20$$

SMath Studio tends to replace these specific values in the calculation of  $F(x)$  as defined above. Redefining  $F(x)$ , using  $xx$  instead of  $x$ , and  $tt$  instead of  $t$ , solves the problem as illustrated below:

$$F(xx) := \int_0^{xx} f(tt) dt$$

Compare the last result with that given by:

$$P(0.2 < X < 1.2) = F(1.2) - F(0.2)$$

Calculating  $F(x)$  for different values of  $x$  now produces the right results:

$$F(0.1) = 0.3624 \quad F(0.5) = 0.8946 \quad F(1.0) = 0.9889 \quad F(1.5) = 0.9988$$


---